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UNIT II

TWO DIMENTIONAL RANDOM VARIABLE

Discrete Random Variable:

If the possible values (outcomes) are finite / countably infinite, then it is called as discrete random variable in 2 Dimentional.

Continuous Random Variable:

If (X, Y) assumes all the possible values in a specified region in XY plane, then it is called as continuous Random Variable in 2D.

Probability Distribution:

Probability mass function:

If (X, Y) is a 2D random variable such that $P\{X = x_i, Y = y_j\} = P_{ij}$ where $i = 1$ to n, $j = 1$ to m, then P_{ij} is called probability mass function with the following conditions.

(i) $P_{ij} \geq 0$

(ii) $\sum_j \sum_i P_{ij} = 1$

It is represented in the form of tabular column given below.

X \ Y	0	1	2	$P\{X = x\} = P_{i*}$
0	P_{00}	P_{01}	P_{02}	$P\{X=0\}$
1	P_{10}	P_{11}	P_{12}	$P\{X=1\}$
2	P_{20}	P_{21}	P_{22}	$P\{X=2\}$
$P\{Y=y\}$ P_{*j}	$P\{Y=0\}$	$P\{Y=1\}$	$P\{Y=2\}$	1

Marginal Probability distribution:

$$P\{X = x_i\} = \sum_j P_{ij} = P_{i*}$$

$$P\{Y = y_j\} = \sum_i P_{ij} = P_{*j}$$

The collection of pairs are (x_i, P_{i*}) is called as marginal probability distribution of X and (y_j, P_{*j}) is called the marginal probability distribution of Y.

The two random variables X & Y are said to be independent if $P_{i*} P_{*j} = P_{ij}$

Cumulative Distribution function (CDF):

If (X, Y) is a bivariate random variable, then its CDF is $F(x,y) = P(X \leq x, Y \leq y) = \sum_{y_j \leq y} \sum_{x_i \leq x} P_{ij}$

Continuous Random Variable in 2 Dimentional:

If (X, Y) is a 2D random variable, the PDF is defined as

$$P\left(x - \frac{dx}{2} \leq x \leq x + \frac{dx}{2}, y - \frac{dy}{2} \leq y \leq y + \frac{dy}{2}\right) = f(x,y)\, dx\, dy$$

Then $f(x,y)$ is called PDF provided the following conditions are satisfied

(i) $f(x,y) \geq 0$

(ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$

Marginal PDF:

Marginal PDF of X.

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

Marginal PDF of Y, $f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$

Cumulative Distribution:

$$F(x, y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(x,y)\, dx\, dy$$

* $F(-\infty, y) = 0$
* $F(x, \infty) = 0$
* $F(-\infty, \infty) = 1$
* $P(a < X < b, Y \leq y) = F(b,y) -$
* $P(X \leq x, c \leq y \leq d) = F(x,d) -$
* $P(a < x < b, c < y < d) = F(b,d) - F(b,c) - F(a,d) + F(a,c)$

NOTE: Two random variables said to be independent.

$$f_X(x)\, f_Y(y) = f(x,y)$$

NOTE :

$$f(x,y) = \frac{\partial^2}{\partial x \partial y} F(x,y)$$

$$P(a<X<b, c<Y<d) = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

$$P(a<X<b) = \int_a^b f_X(x)\,dx = \int_a^b \left(\int_{-\infty}^{\infty} f(x,y)\,dy \right) dx$$

$$P(c<Y<d) = \int_c^d f_Y(y)\,dy = \int_c^d \left(\int_{-\infty}^{\infty} f(x,y)\,dx \right) dy$$

Conditional Probability Distribution:

(i) $$P(X=x_i / Y=y_j) = \frac{P\{X=x_i, Y=y_j\}}{P\{Y=y_j\}}$$

$$= \frac{P_{ij}}{P_{*j}}$$

IIIly $$P(Y=y_j / X=x_i) = \frac{P\{Y=y_j, X=x_i\}}{P\{X=x_i\}} = \frac{P\{X=x_i, Y=y_j\}}{P\{X=x_i\}}$$

$$= \frac{P_{ij}}{P_{i*}}$$

(ii) $$f_{X|Y}(x/y) = \frac{f(x,y)}{f_Y(y)}$$

(iii) $$f_{Y|X}(y/x) = \frac{f(x,y)}{f_X(x)}$$

Prob:

i) The joint PDF of (X,Y) is given by $P(x,y) = k(2x+3y)$; $x = 0,1,2$; $y = 1,2,3$. (ii) Find all the marginal and (iii) conditional distributions. (iv) Find $P(X \geq 1, Y \leq 2)$. Find (v) probability distribution of $X+Y$.

$P(X+Y \leq 4)$.

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$k(2x+3y)$

$x \backslash y$	1	2	3	$P\{X=x_i\}$
0	3k	6k	9k	18k
1	5k	8k	11k	24k
2	7k	10k	13k	30k
$P\{Y=y_j\}$	15k	24k	33k	72k

WKT, $\sum_i \sum_j P_{ij} = 1$

$72k = 1$

$\boxed{k = 1/72}$

(i)

$x \backslash y$	1	2	3	$P\{X=x_i\}$
0	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{8}$	$\frac{1}{4}$
1	$\frac{5}{72}$	$\frac{1}{9}$	$\frac{11}{72}$	$\frac{1}{3}$
2	$\frac{7}{72}$	$\frac{5}{36}$	$\frac{13}{72}$	$\frac{5}{12}$
$P\{Y=y_j\}$	$\frac{5}{24}$	$\frac{1}{3}$	$\frac{11}{24}$	1

(ii) Marginal distribution of X:

$P\{X=x_i\} = \sum_j P_{ij} = P_{i*}$

x	$P\{X=x_i\}$
0	$1/4$
1	$1/3$
2	$5/12$
Total	1

Marginal Distribution of Y

$P\{Y=y_j\} = \sum_i P_{ij} = P_{*j}$

y	$P\{Y=y_j\}$
1	$5/24$
2	$1/3$
3	$11/24$
Total	1

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(iii) Conditional Distribution of x/y :

$$P\{x=x_i/y=y_j\} = \frac{P\{x=x_i, Y=y_j\}}{P\{Y=y_j\}}$$

$$= \frac{P_{ij}}{P_{*j}}$$

Let $P\{X=x_i/Y=1\} = \frac{P_{i1}}{P_{*1}} = \frac{P_{i1}}{P\{Y=1\}}$ $P\{X=x_i/Y=2\} = \frac{P_{i2}}{P_{*2}} = \frac{P_{i2}}{P\{Y=2\}}$ $P\{X=x_i/Y=3\} = \frac{P_{i3}}{P_{*3}}$

x	$P\{X=x_i/Y=1\}$	
0	$\frac{P_{01}}{P_{*1}} = \frac{1/24}{5/24}$	$= \frac{1}{5}$
1	$\frac{P_{11}}{P_{*1}} = \frac{5/72}{5/24}$	$= \frac{1}{3}$
2	$\frac{P_{21}}{P_{*1}} = \frac{7/72}{5/24}$	$= \frac{7}{15}$

x	$P\{X=x_i/Y=2\}$
0	1/4
1	1/3
2	5/12

x	$P\{X=x_i/Y=3\}$
0	3/11
1	1/3
2	13/33

$$P\{Y=y_j/X=x_i\} = \frac{P\{X=x_i, Y=y_j\}}{P\{X=x_i\}} = \frac{P_{ij}}{P_{i*}}$$

$P\{Y=y_j/X=0\} = \frac{P_{0j}}{P_{0*}} = \frac{P_{0j}}{P\{X=0\}}$ $P\{Y=y_j/X=1\} = \frac{P_{1j}}{P_{1*}} = \frac{P_{1j}}{P\{X=1\}}$ $P\{Y=y_j/X=2\} = \frac{P_{2j}}{P_{2*}}$

Y	$P\{Y=y_j/X=0\}$
1	1/6
2	1/3
3	1/2

Y	$P\{Y=y_j/X=1\}$
1	5/24
2	1/3
3	11/24

Y	$P\{Y=y_j/X=2\}$
1	7/30
2	1/3
3	13/30

(iv) $P(X \geq 1, Y \leq 2) = P(X=1, Y=2) + P(X=1, Y=1) + P(X=2, Y=2) + P(X=2, Y=1)$

$$= \frac{5}{72} + \frac{1}{9} + \frac{5}{36} + \frac{7}{72} = \frac{5}{12}$$

(v)

X+Y	P	
1	1/24	1/24
2	$\frac{1}{12} + \frac{5}{72}$	11/72
3	$\frac{1}{8} + \frac{1}{9} + \frac{7}{72}$	1/3
4	$\frac{11}{72} + \frac{5}{36}$	7/24
5	13/72	13/72

2) For 2D PD of (X,Y) given below, find (i) $P(X \le 1)$, (ii) $P(Y \le 3)$, (iii) $P(X \le 1, Y \le 3)$; (iv) $P(X \le 1 / Y \le 3)$, (v) $P(Y \le 3 / X \le 1)$, (vi) $P(X+Y) \le 4)$

X \ Y	1	2	3	4	5	6	$P\{X=x_i\}$
0	0	0	1/32	2/32	2/32	3/32	1/4
1	1/16	1/16	1/8	1/8	1/8	1/8	5/8
2	1/32	1/32	1/64	1/64	0	2/64	1/8
$P\{Y=y_j\}$	3/32	3/32	11/64	13/64	3/16	1/4	1

(i) $P(X \le 1) = P(X=0) + P(X=1) = \frac{1}{4} + \frac{5}{8} = \frac{7}{8}$

(ii) $P(Y \le 3) = P(Y=1) + P(Y=2) + P(Y=3) = \frac{3}{32} + \frac{3}{32} + \frac{11}{64} = \frac{23}{64}$

(0,1) (1,2,3)

(iii) $P(X \le 1, Y \le 3) = P(X=0, Y=1) + P(X=0, Y=2) + P(X=0, Y=3) + P(X=1, Y=1) + P(X=1, Y=2) + P(X=1, Y=3)$

$= 0 + 0 + \frac{1}{32} + \frac{1}{16} + \frac{1}{16} + \frac{1}{8}$

$= \frac{9}{32}$

~~(iv) $P(X \le 1 / Y = 1) = \frac{P_{i1}}{P_{*1}}$; $P(X \le 1 / Y=2) = \frac{P_{i2}}{P_{*2}}$; $P(X \le 1 / Y=3) = \frac{P_{i3}}{P_{*3}}$~~

X	$P(X \le 1/Y=1)$	X	$P(X \le 1/Y=2)$	X	$P(X \le 1/Y=3)$
0	0	0	0	0	2/11
1	2/3	1	2/3	1	

(iv) $P(X \le 1 / Y \le 3) = \frac{P(X \le 1, Y \le 3)}{P(Y \le 3)} = \frac{9/32}{23/64} = \frac{18}{23}$

(v) $P(Y \le 3 / X \le 1) = \frac{P(Y \le 3, X \le 1)}{P(X \le 1)} = \frac{9/32}{7/8} = \frac{9}{28}$

(vi) $P(X+Y \le 4)$

$X+Y$	P	
1	0	$= 0$
2	$0 + \frac{1}{16}$	$= \frac{1}{16}$
3	$\frac{1}{32} + \frac{1}{16} + \frac{1}{32}$	$= \frac{1}{8}$
4	$\frac{2}{32} + \frac{1}{8} + \frac{1}{32}$	$= \frac{7}{32}$

$P(X+Y \le 4) = 13/32$

3) The joint PDF of (X,Y) is given by $f(x,y) = c(2x+y)$ $0 \le x \le 2$, $0 \le y \le 3$ where X & Y are assumed all integers between X & Y.
(i) Find c. (ii) $P(X \ge 1, Y \le 2)$

Soln:

$X \backslash Y$	0	1	2	3	$P\{X=x_i\}$
0	0	c	2c	3c	6c
1	2c	3c	4c	5c	14c
2	4c	5c	6c	7c	22c
$P\{Y=y_j\}$	6c	9c	12c	15c	42c

$$\sum_i \sum_j P_{ij} = 1$$

$$42c = 1$$

$$\boxed{c = 1/42}$$

$X \backslash Y$	0	1	2	3	$P\{X=x_i\}$
0	0	1/42	1/21	1/14	2/21
1	1/21	1/14	2/21	5/42	1/3
2	2/21	5/42	1/7	1/6	11/21
$P\{Y=y_j\}$	1/7	3/14	2/7	5/14	1

$$P(x\geq 1, y\leq 2) = P(x=1, y=0) + P(x=1, y=1) + P(x=1, y=2) + P(x=2, y=0) + P(x=2, y=1) + P(x=2, y=2)$$

$$= \frac{1}{21} + \frac{1}{14} + \frac{2}{21} + \frac{2}{21} + \frac{5}{42} + \frac{1}{7}$$

$$P(x\geq 1, y\leq 2) = \frac{4}{7}$$

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4) A machine is used for a particular job in the forenoon and for the different job in the afternoon. The joint pdf of (x, y) is given in following table. Examine x and y are independent or not.

x \ y	0	1	2	$P\{x=x_i\}$ P_{*j}
0	0.1	0.04	0.06	0.2 P_{0j}
1	0.2	0.08	0.12	0.4 P_{1j}
2	0.2	0.08	0.12	0.4 P_{2j}
P_{*i} $P\{y=y_i\}$	0.5 P_{i0}	0.2 P_{i1}	0.3 P_{i2}	1

$$P_{i*} \times P_{*j} = P_{ij}$$

let

$P_{0*} \times P_{*0} = 0.1 = P_{00}$

$P_{0*} \times P_{*1} = 0.04 = P_{01}$

$P_{0*} \times P_{*2} = 0.06 = P_{02}$

$P_{1*} \times P_{*0} = 0.2 = P_{10}$

$P_{1*} \times P_{*1} = 0.08 = P_{11}$

$P_{1*} \times P_{*2} = 0.12 = P_{12}$

$P_{2*} \times P_{*0} = 0.2 = P_{20}$

$P_{2*} \times P_{*1} = 0.8 = P_{21}$

$P_{2*} \times P_{*2} = 0.12 = P_{22}$

$\therefore$ x and y are independent

5) Given $f(x_1, x_2) = \frac{x_1+x_2}{21}$, $x_1 = 1,2,3;\ x_2 = 1,2$

find marginal & ~~conditional~~ distributions of X_1 & X_2

~~Contd:~~ & $E(XY)$

6) If the joint pdf of a 2D random variable is given by

$$f(x,y) = \begin{cases} K(6-x-y), & 0<x<2,\ 2<y<4 \\ 0, & \text{elsewhere.} \end{cases}$$

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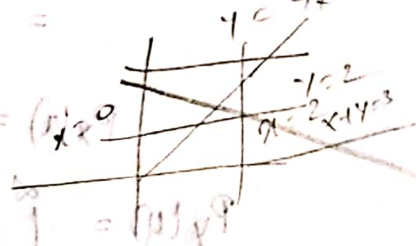
(i) Find K. (ii) $P(X<1, Y<3)$ (iii) Find marginal densities.

(iv) $P(X+Y<3)$ (v) $P(X<1 / Y<3)$

(i) To find k,

we know that



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$K\int_2^4\int_0^2 (6-x-y)\,dx\,dy = 1$$

$$K\int_2^4 \left[6x - \frac{x^2}{2} - xy\right]_0^2 dy = 1$$

$$K\int_2^4 [12-2-2y]\,dy = 1$$

$$K\left[10y - \frac{2y^2}{2}\right]_2^4 = 1$$

$$K[40-16-(20-4)] = 1$$

$$K[40-16-16] = 1$$

$$8K = 1$$

$$\boxed{K = 1/8} \Rightarrow f(x,y) = \begin{cases} \frac{(6-x-y)}{8}, & 0<x<2,\ 2<y<4 \\ 0, & \text{elsewhere.} \end{cases}$$

(ii) $P(X<1, Y<3) = \frac{1}{8}\int_2^3\int_0^1 (6-x-y)\,dx\,dy = \frac{1}{8}\int_2^3 \left[6x - \frac{x^2}{2} - xy\right]_0^1 dy$

$$= \frac{1}{8}\int_2^3 \left(6 - \frac{1}{2} - y\right) dy = \frac{1}{8}\left[6y - \frac{y}{2} - \frac{y^2}{2}\right]_2^3$$

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$$= \frac{1}{8}\left[18 - \frac{3}{2} - \frac{9}{2} - (12 - 1 - 2)\right]$$

$$= \frac{1}{8}\left[18 - 9 - 6\right]$$

$$P(X<1, Y<3) = \frac{3}{8}$$

(iii) $$P_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \frac{1}{8}\int_2^4 (6-x-y)\,dy = \frac{1}{8}\left[6y - xy - \frac{y^2}{2}\right]_2^4$$

$$= \frac{1}{8}\left[24 - 4x - 8 - (12 - 2x - 2)\right]$$

$$= \frac{1}{8}\left[16 - 4x - 10 + 2x\right]$$

$$= \frac{6-2x}{8}$$

$$P_X(x) = \frac{3-x}{4}, \quad 0<x<2$$

$$P_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \frac{1}{8}\int_0^2 (6-x-y)\,dx = \frac{1}{8}\left[6x - \frac{x^2}{2} - xy\right]_{x=0}^{2}$$

$$= \frac{1}{8}\left[12 - 2 - 2y - (0)\right]$$

$$= \frac{10-2y}{8}$$

$$P_Y(y) = \frac{5-y}{4}, \quad 2<y<4$$

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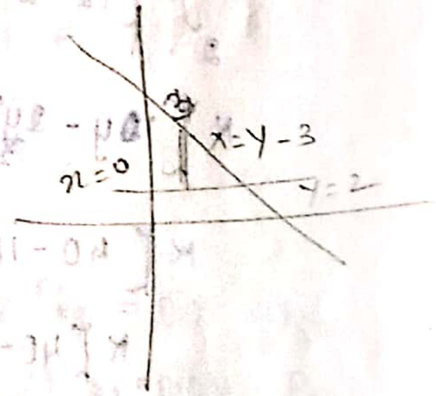
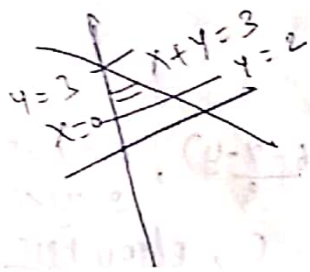
(iv) $$P(X+Y<3) = \int_2^3\int_0^{3-y} f(x,y)\,dx\,dy$$



$$= \int_2^3\int_0^{3-y} \frac{1}{8}(6-x-y)\,dx\,dy$$



$$= \frac{1}{8}\int_2^3 \left[6x - \frac{x^2}{2} - xy\right]_0^{3-y} dy$$

$$= \frac{1}{8}\int_2^3 \left[6(3-y) - \frac{(3-y)^2}{2} - (3-y)y\right] dy$$

$$= \frac{1}{8}\int_2^3 \left[18 - 6y - \frac{1}{2}(9+y^2-6y) - 3y + y^2\right] dy$$

$$= \frac{1}{8}\left[18y - \frac{6y^2}{2} - \frac{9y}{2} - \frac{y^3}{6} + \frac{6y^2}{\cancel{4}_2} - \frac{3y^2}{2} + \frac{y^3}{3}\right]_2^3$$

$$= \frac{1}{8}\left[54 - 27 - \frac{27}{2} - \frac{27}{6} + \cancel{\frac{27}{2}} - \cancel{\frac{27}{2}} + \frac{27}{3} - \left(36 - 12 - 9 - \frac{8}{6} + \cancel{\frac{12}{2}} - \cancel{\frac{12}{2}} + \frac{8}{3}\right)\right]$$

$$P(X+Y<3) = \frac{5}{24} \;//$$

(v) $P(X<1 / Y<3) = \dfrac{P(X<1, Y<3)}{P(Y<3)}$

$$P(Y<3) = \int_2^3 f_Y(y)\,dy = \frac{1}{8}\int_2^3 (10-2y)\,dy$$

$$= \frac{1}{8}\left[10y - \frac{2y^2}{2}\right]_2^3$$

$$= \frac{1}{8}\left[30 - 9 - 20 + 4\right]$$

$$P(Y<3) = \frac{5}{8}$$

$$P(X<1 / Y<3) = \frac{3/8}{5/8} = \frac{3}{5} \;//$$

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8) Joint pdf of (X, Y) is given by $f(x,y) = \begin{cases} kxye^{-(x^2+y^2)}, & x>0, y>0 \\ 0 & \text{otherwise.} \end{cases}$

(i) Find k (ii) Prove X & Y are independent
(iii) ~~Find~~ ~~conditional densities~~ ~~(iv) $P(Y<1)$~~ ~~(v) $P(X \le Y)$~~

$v = e^{-x^2}$
$u = x$, $v_1 = e^{-x}$

Wkt,
(i)
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$k\int_{0}^{\infty} y \int_{0}^{\infty} x e^{-(x^2+y^2)}\,dx\,dy = 1$$

$$k\int_{0}^{\infty} y e^{-y^2}\,dy \int_{0}^{\infty} x e^{-x^2}\,dx = 1$$

put $x^2 = u$
$2x\,dx = du$
$x\,dx = du/2$
when $x = 0$, $u = 0$
$x = \infty$, $u = \infty$

put $y^2 = v$
$2y\,dy = dv$
$y\,dy = dv/2$
when $y = 0$, $v = 0$
$y = \infty$, $v = \infty$

$$\frac{k}{4}\int_{0}^{\infty} e^{-u}\,du \int_{0}^{\infty} e^{-v}\,dv = 1$$

$$k\,[-e^{-u}]_0^{\infty}\,[-e^{-v}]_0^{\infty} = 4$$

$$k\,[1]\,[1] = 4$$

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$\boxed{K=4}$

$$\therefore f(x,y) = \begin{cases} 4xye^{-(x^2+y^2)}, & x>0, y>0 \\ 0, & \text{otherwise} \end{cases}$$

(ii) Marginal densities of x:

$$f_X(x) = \int_0^\infty f(x,y)dy = 4\int_0^\infty xye^{-(x^2+y^2)}dy$$

$$= 4xe^{-x^2}\int_0^\infty ye^{-y^2}dy$$

put $y^2 = v$

$2ydy = dv$; when $y=0, v=0$

$ydy = dv/2$; $y=\infty, v=\infty$

$$f_X(x) = 4\frac{xe^{-x^2}}{2}\int_0^\infty e^{-v}dv$$

$$= 2xe^{-x^2}[-e^{-v}]_0^\infty$$

$$f_X(x) = 2xe^{-x^2}, \ x>0$$

$$f_Y(y) = \int_0^\infty f(x,y)dx = 4ye^{-y^2}\int_0^\infty xe^{-x^2}dx$$

put $x^2 = u$

$2xdx = du$; when $x=0, u=0$

$xdx = du/2$; $x=\infty, u=\infty$

$$f_Y(y) = 4\frac{ye^{-y^2}}{2}\int_0^\infty e^{-u}du$$

$$= 2ye^{-y^2}[-e^{-u}]_0^\infty$$

$$f_Y(y) = 2ye^{-y^2}, \ y>0$$

$$f_X(x).f_Y(y) = 2xe^{-x^2} \cdot 2ye^{-y^2}, \ x>0, y>0$$

$$= 4xye^{-(x^2+y^2)}, \ x>0, y>0$$

$$f_X(x).f_Y(y) = f(x,y)$$

$\therefore X, Y$ are independent.

(iii) Conditional densities:

$$f_{X/Y}(x/y) = \frac{f(x,y)}{f(y)} = \frac{4xye^{-(x^2+y^2)}}{2ye^{-y^2}}$$

9) If joint pdf of (x,y) is given by $f(x,y) = \begin{cases} k(1-x-y), & 0<x, y<\frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

(i) Find k

Wkt,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$k\int_0^{1/2}\int_0^{1/2} (1-x-y)\,dx\,dy = 1$$

$$k\int_0^{1/2}\left[x-\frac{x^2}{2}-xy\right]_{x=0}^{1/2} dy = 1$$

$$k\int_0^{1/2}\left(\frac{1}{2}-\frac{1}{8}-\frac{y}{2}\right)dy = 1$$

$$k\left[\frac{y}{2}-\frac{y}{8}-\frac{y^2}{4}\right]_0^{1/2} = 1$$

$$k\left[\frac{1}{4}-\frac{1}{16}-\frac{1}{16}\right] = 1$$

$$k\left(\frac{1}{8}\right) = 1$$

$$\boxed{k=8} \Rightarrow f(x,y) = \begin{cases} 8(1-x-y), & 0<x, y<\frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

10) Find k if $f(x,y) = \begin{cases} k(1-x)(1-y), & 0<x, y<1 \\ 0, & \text{otherwise} \end{cases}$

$$K\int_0^1 (1-y)\int_0^1 (1-x)\,dx\,dy = 1$$

$$K\int_0^1 (1-y)\left[x-\frac{x^2}{2}\right]_0^1 dy = 1$$

$$K\int_0^1 (1-y)\left[1-\frac{1}{2}\right]dy = 1$$

$$\frac{K}{2}\left[y-\frac{y^2}{2}\right]_0^1 = 1$$

$$\frac{K}{2}\left[1-\frac{1}{2}\right] = 1$$

$$\frac{K}{4} = 1$$

$$\boxed{K=4}$$

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